



POLITÉCNICA

“Ingeniamos el futuro”

Inversor CMOS

Créditos de las transparencias:

- Digital Integrated Circuits. Second Edition. Jan M. Rabaey, Anantha Chandrakasan, and Borivoje Nikolic
- Asignatura de David Harris en el Harvey Mudd College
- EGE535 Low Power VLSI Design, Damu Radhakrishnan, State University of New York
- Elaboración Propia

Clase de hoy

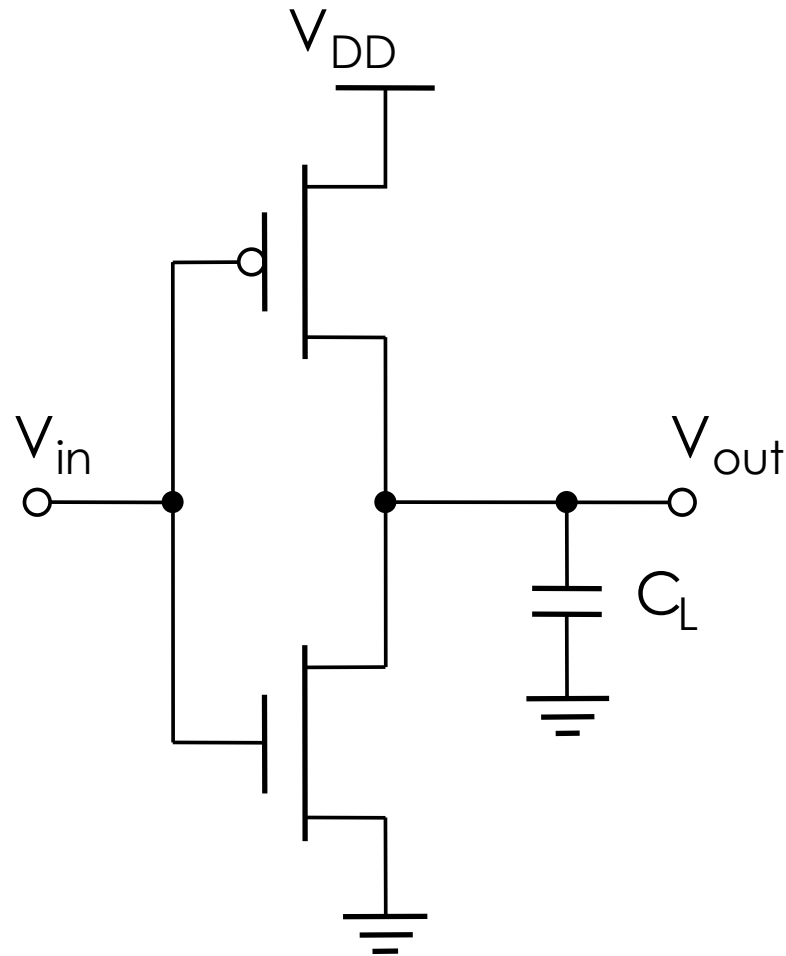
- Inversor CMOS
 - Comportamiento en estática
 - Métricas de calidad
 - Comportamiento en dinámica
 - Otros estilos lógicos

Bibliografía:

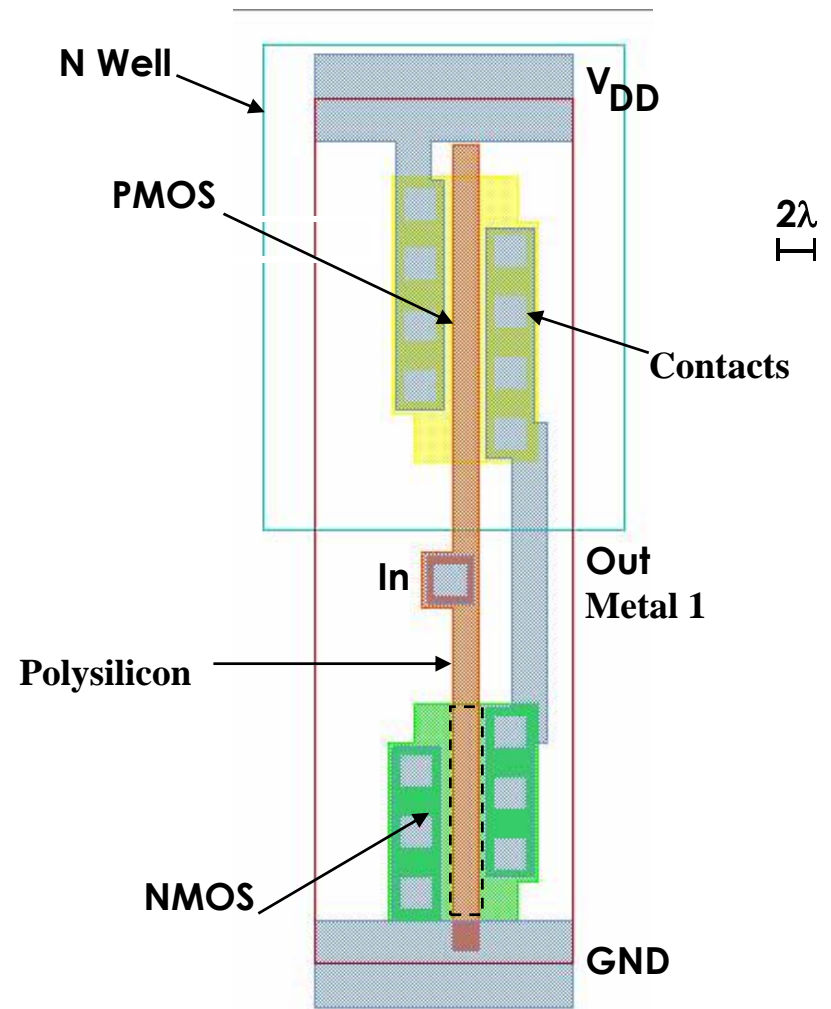
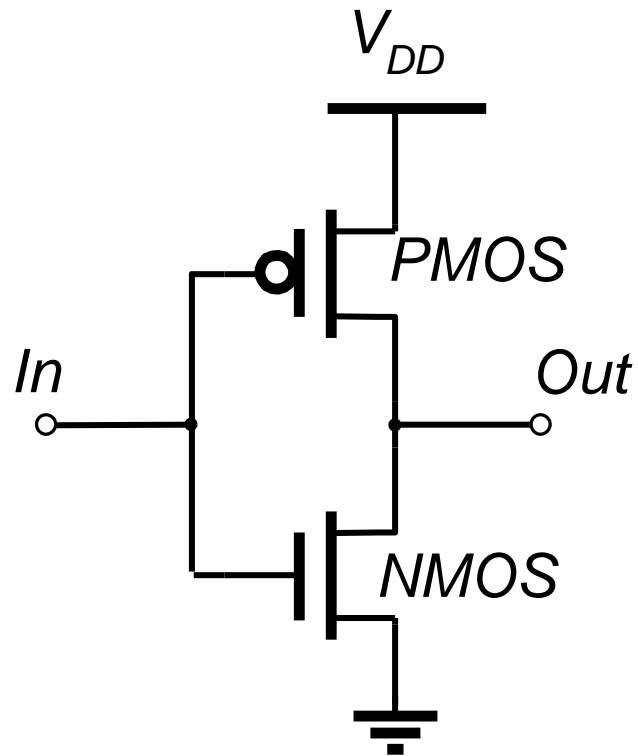
Capítulo 2 Weste-Eshraghian

Capítulo 5 Rabaey

The CMOS Inverter: A First Glance



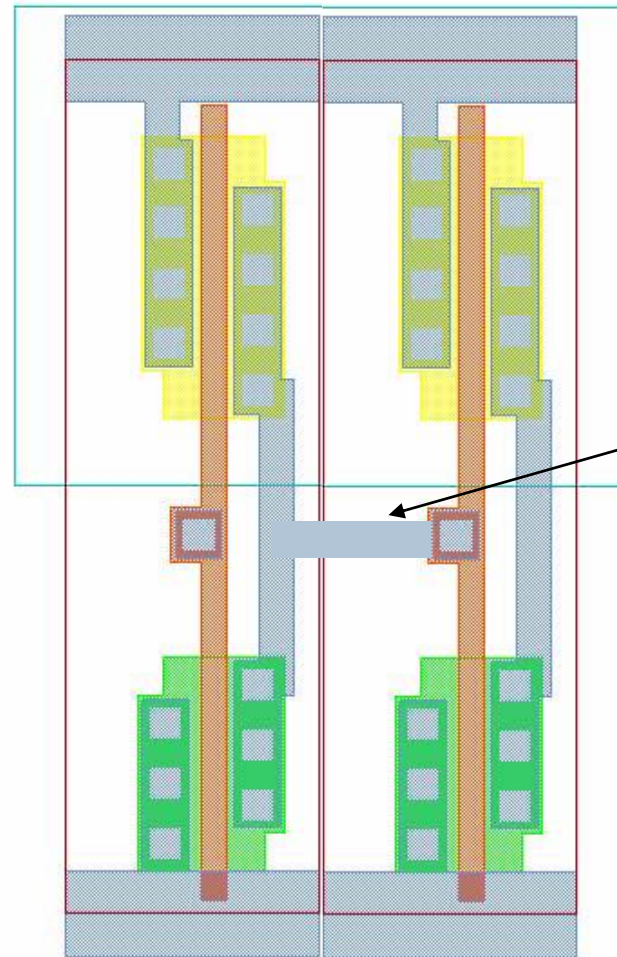
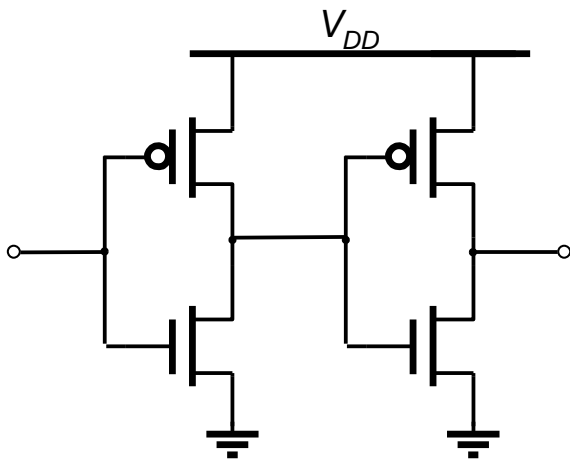
CMOS Inverter



Two Inverters

Share power and ground

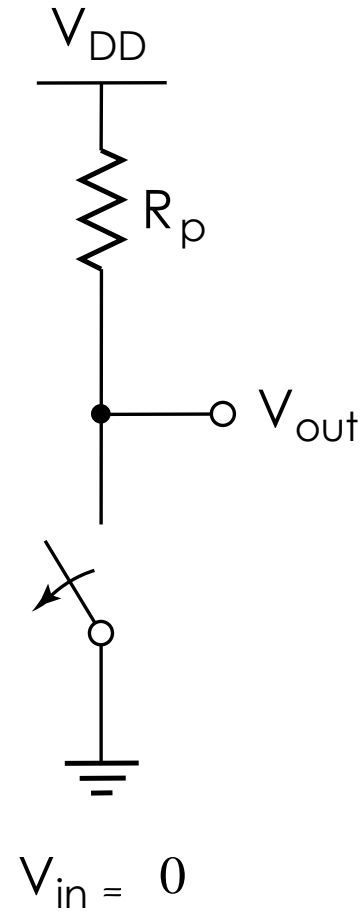
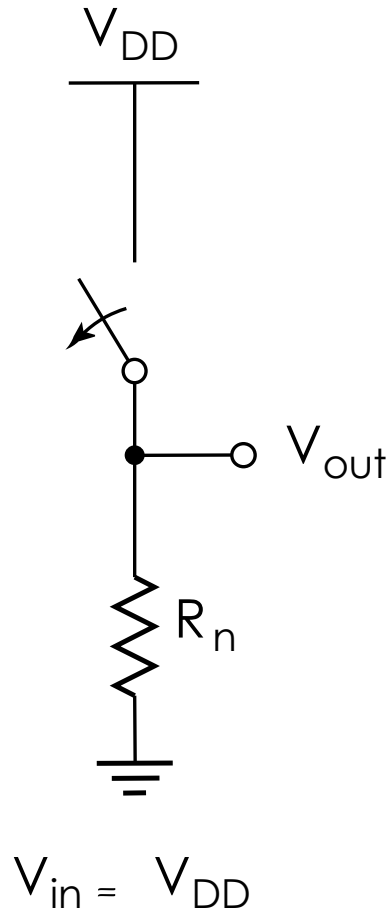
Abut cells



Connect in Metal

CMOS Inverter

First-Order DC Analysis

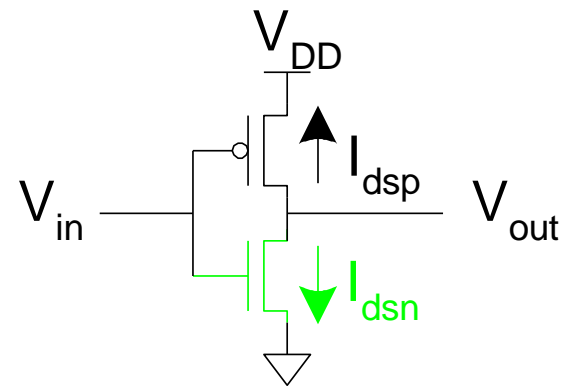


Transistor Operation

- Current depends on region of transistor behavior
- For what V_{in} and V_{out} are nMOS and pMOS in
 - Cutoff?
 - Linear?
 - Saturation?

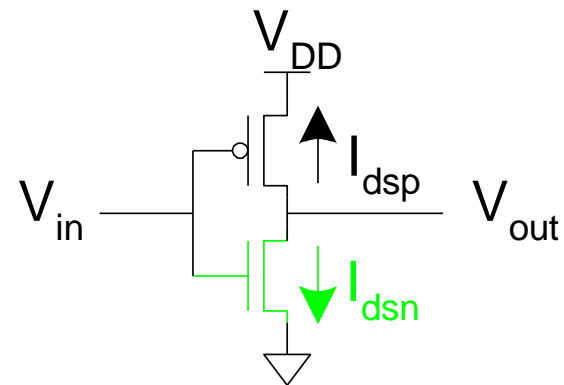
nMOS Operation

Cutoff	Linear	Saturated
$V_{gsn} <$	$V_{gsn} >$	$V_{gsn} >$
	$V_{dsn} <$	$V_{dsn} >$



nMOS Operation

Cutoff	Linear	Saturated
$V_{gsn} < V_{tn}$	$V_{gsn} > V_{tn}$	$V_{gsn} > V_{tn}$
	$V_{dsn} < V_{gsn} - V_{tn}$	$V_{dsn} > V_{gsn} - V_{tn}$

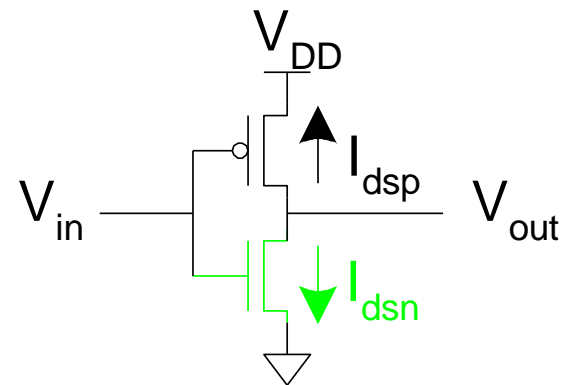


nMOS Operation

Cutoff	Linear	Saturated
$V_{gsn} < V_{tn}$	$V_{gsn} > V_{tn}$	$V_{gsn} > V_{tn}$
	$V_{dsn} < V_{gsn} - V_{tn}$	$V_{dsn} > V_{gsn} - V_{tn}$

$$V_{gsn} = V_{in}$$

$$V_{dsn} = V_{out}$$

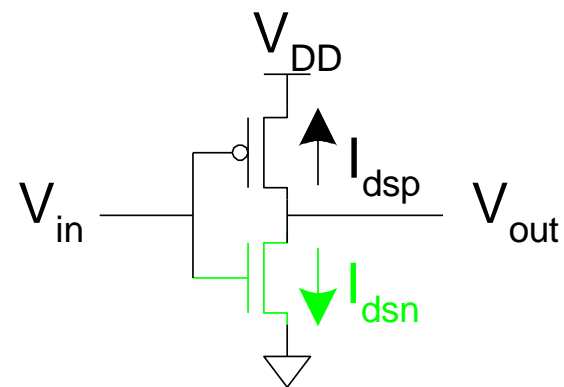


nMOS Operation

Cutoff	Linear	Saturated
$V_{gsn} < V_{tn}$ $V_{in} < V_{tn}$	$V_{gsn} > V_{tn}$ $V_{in} > V_{tn}$ $V_{dsn} < V_{gsn} - V_{tn}$ $V_{out} < V_{in} - V_{tn}$	$V_{gsn} > V_{tn}$ $V_{in} > V_{tn}$ $V_{dsn} > V_{gsn} - V_{tn}$ $V_{out} > V_{in} - V_{tn}$

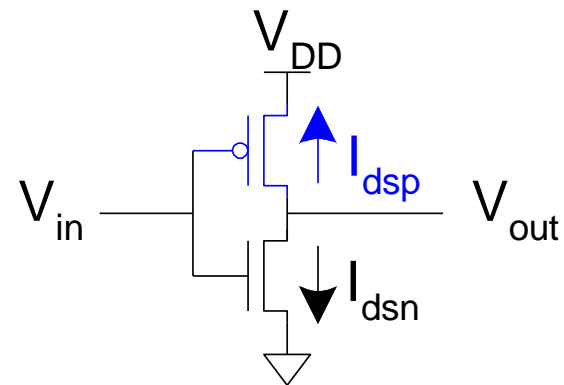
$$V_{gsn} = V_{in}$$

$$V_{dsn} = V_{out}$$



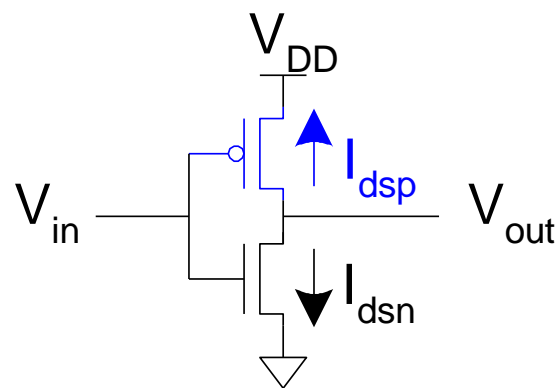
pMOS Operation

Cutoff	Linear	Saturated
$V_{gsp} >$	$V_{gsp} <$	$V_{gsp} <$
	$V_{dsp} >$	$V_{dsp} <$



pMOS Operation

Cutoff	Linear	Saturated
$V_{gsp} > V_{tp}$	$V_{gsp} < V_{tp}$	$V_{gsp} < V_{tp}$
	$V_{dsp} > V_{gsp} - V_{tp}$	$V_{dsp} < V_{gsp} - V_{tp}$



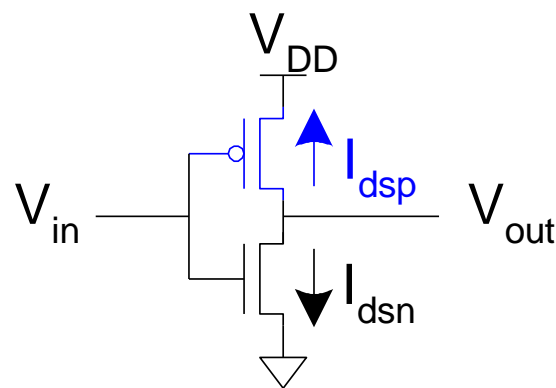
pMOS Operation

Cutoff	Linear	Saturated
$V_{gsp} > V_{tp}$	$V_{gsp} < V_{tp}$	$V_{gsp} < V_{tp}$
	$V_{dsp} > V_{gsp} - V_{tp}$	$V_{dsp} < V_{gsp} - V_{tp}$

$$V_{gsp} = V_{in} - V_{DD}$$

$$V_{tp} < 0$$

$$V_{dsp} = V_{out} - V_{DD}$$



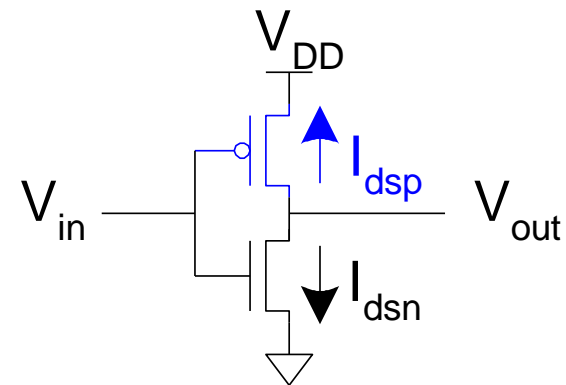
pMOS Operation

Cutoff	Linear	Saturated
$V_{gsp} > V_{tp}$ $V_{in} > V_{DD} + V_{tp}$	$V_{gsp} < V_{tp}$ $V_{in} < V_{DD} + V_{tp}$ $V_{dsp} > V_{gsp} - V_{tp}$ $V_{out} > V_{in} - V_{tp}$	$V_{gsp} < V_{tp}$ $V_{in} < V_{DD} + V_{tp}$ $V_{dsp} < V_{gsp} - V_{tp}$ $V_{out} < V_{in} - V_{tp}$

$$V_{gsp} = V_{in} - V_{DD}$$

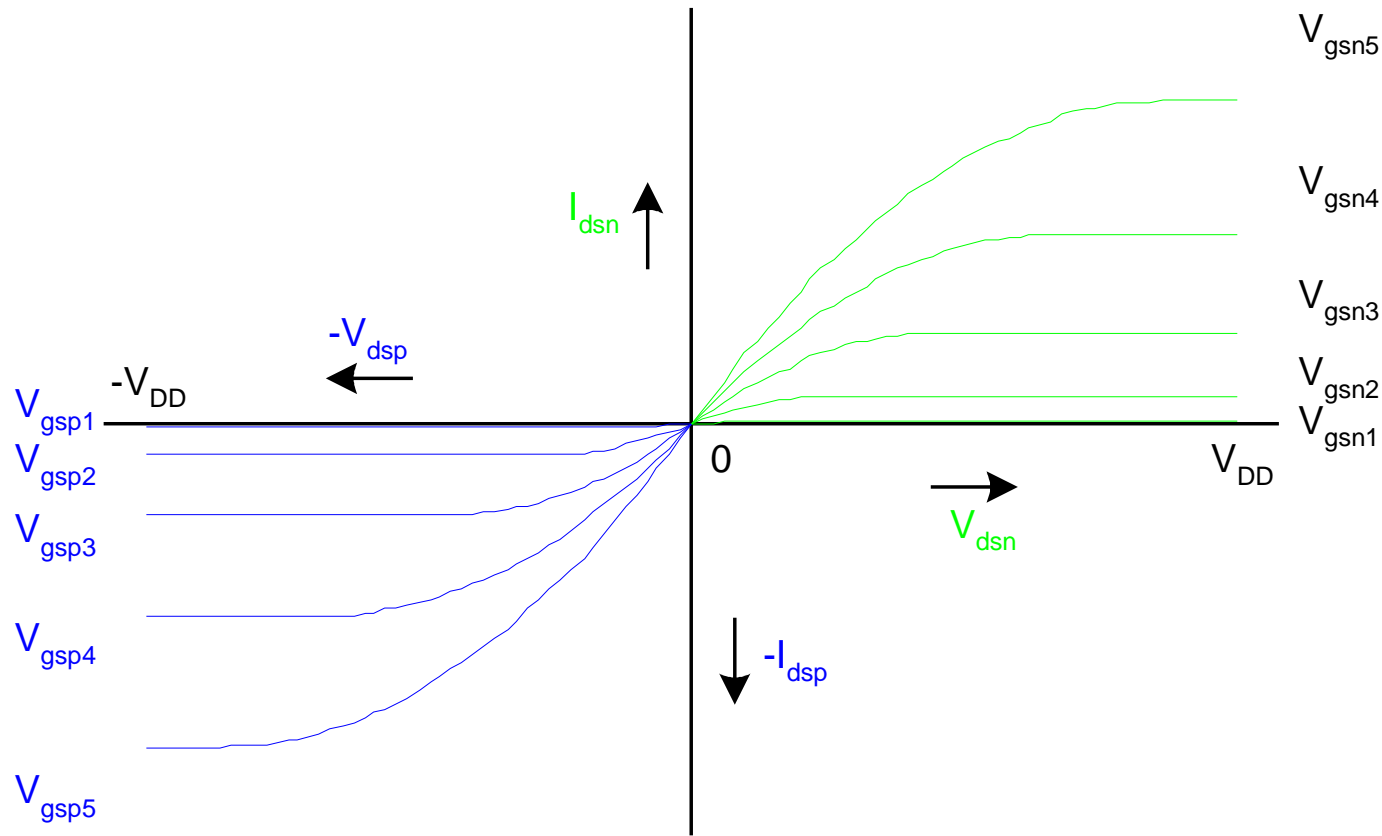
$$V_{tp} < 0$$

$$V_{dsp} = V_{out} - V_{DD}$$

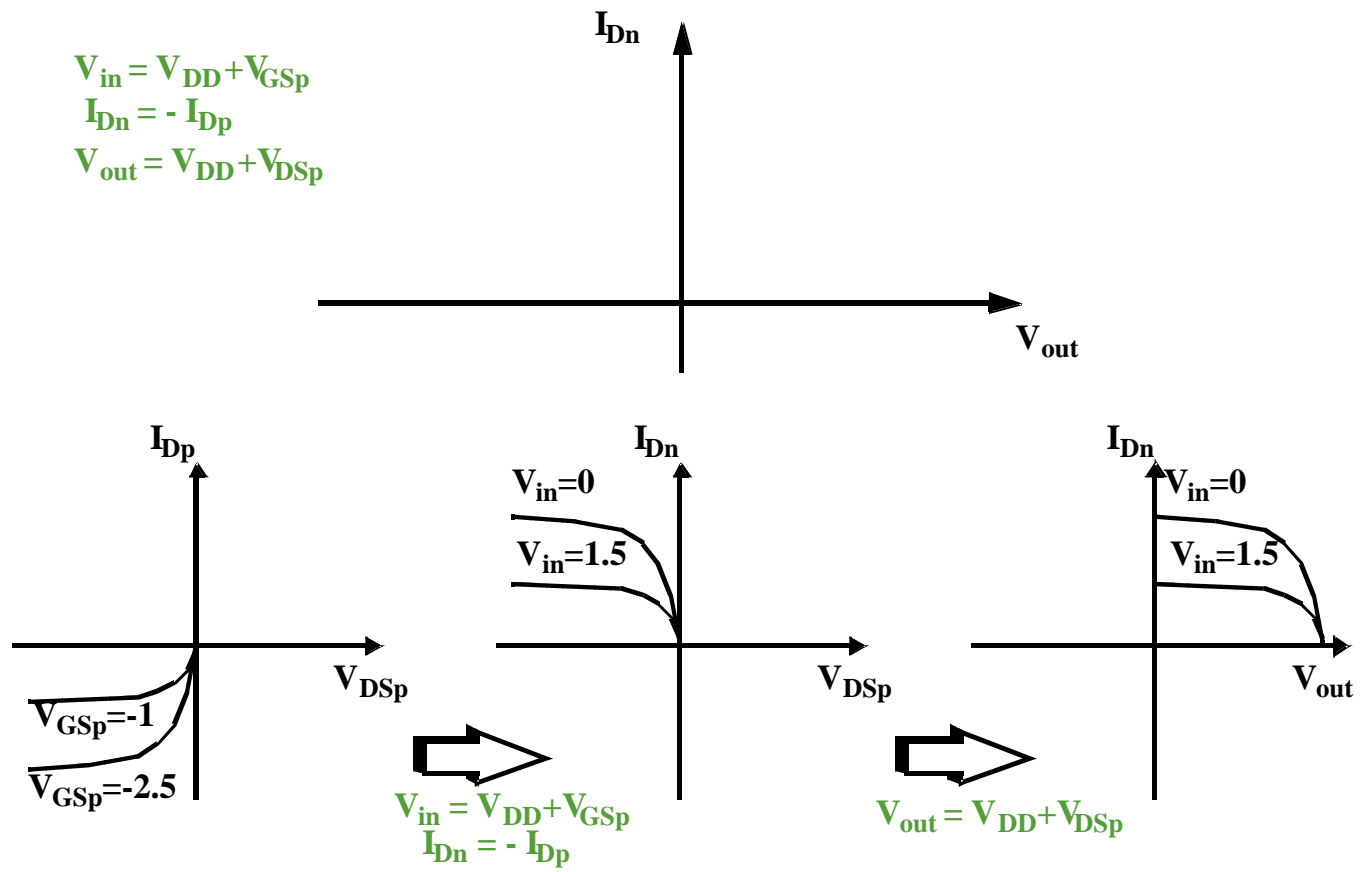


I-V Characteristics

- Make pMOS is wider than nMOS such that $\beta_n = \beta_p$

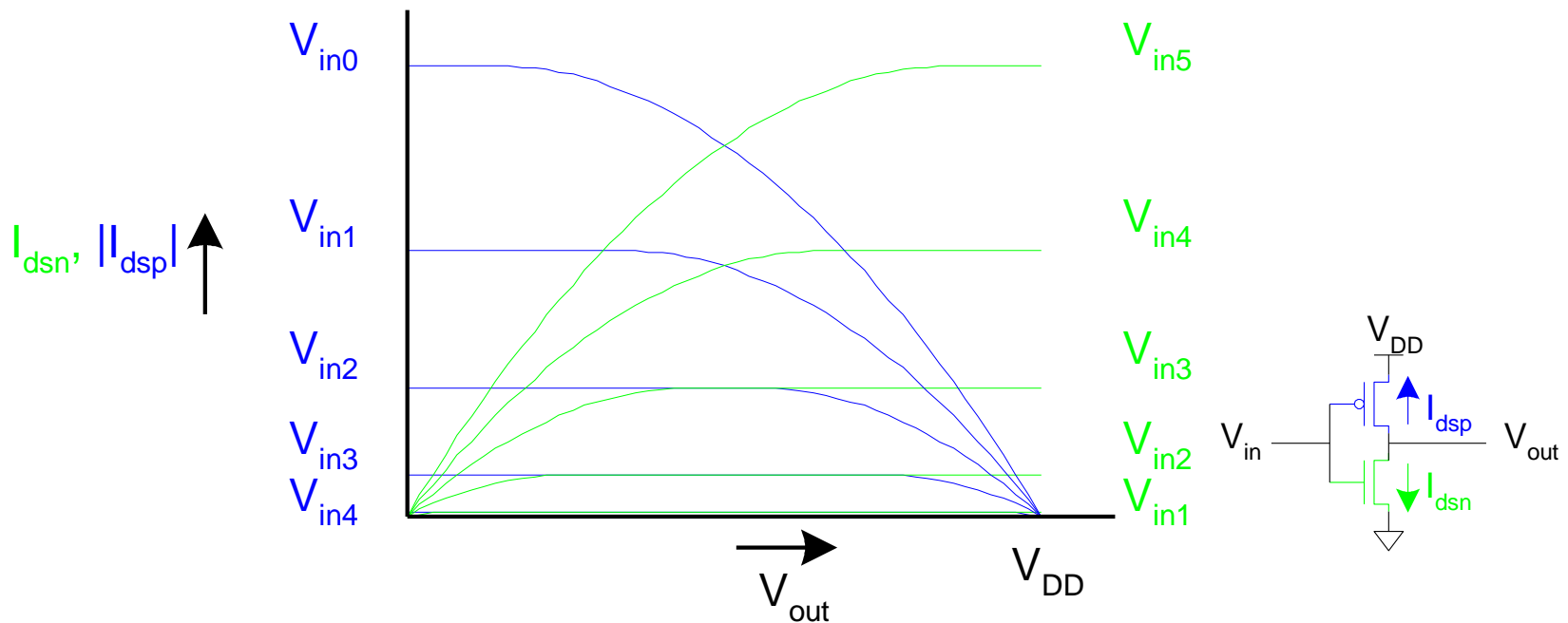


PMOS Load Lines



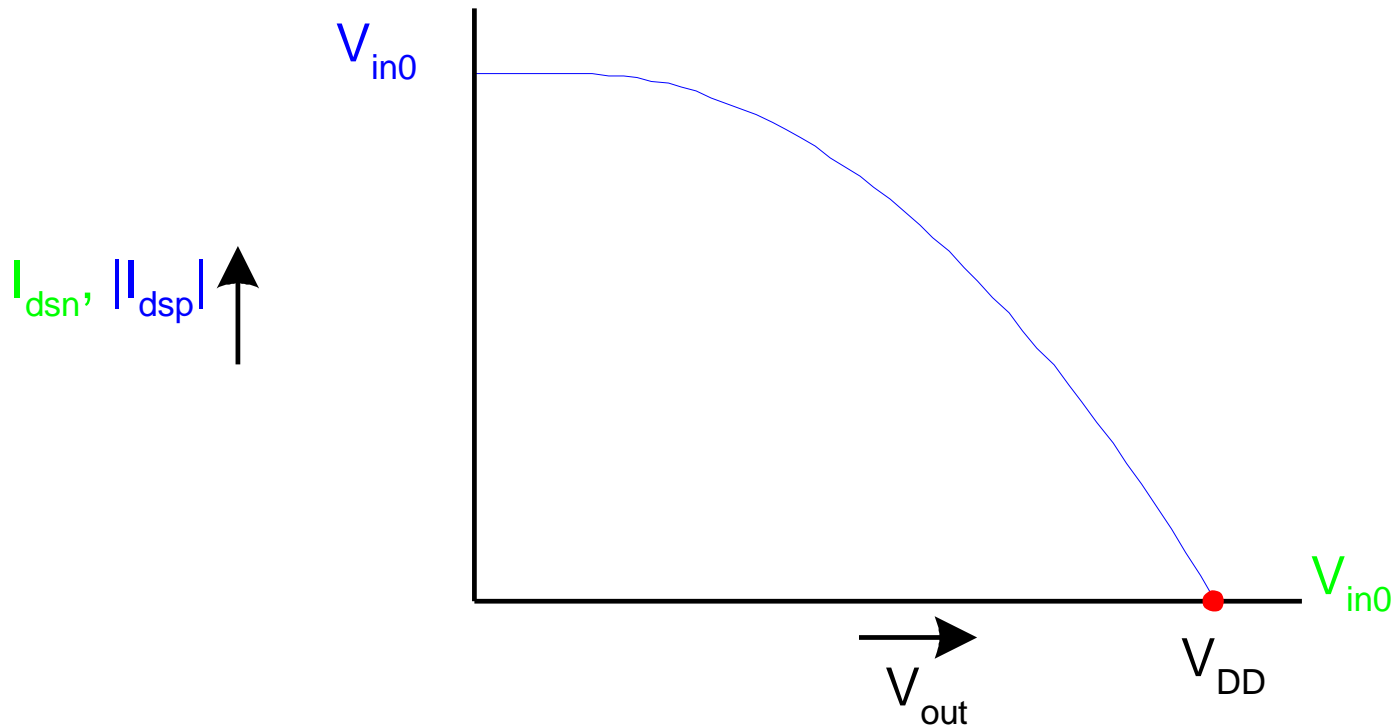
Load Line Analysis

- For a given V_{in} :
 - Plot I_{dsn} , I_{dsp} vs. V_{out}
 - V_{out} must be where |currents| are equal in



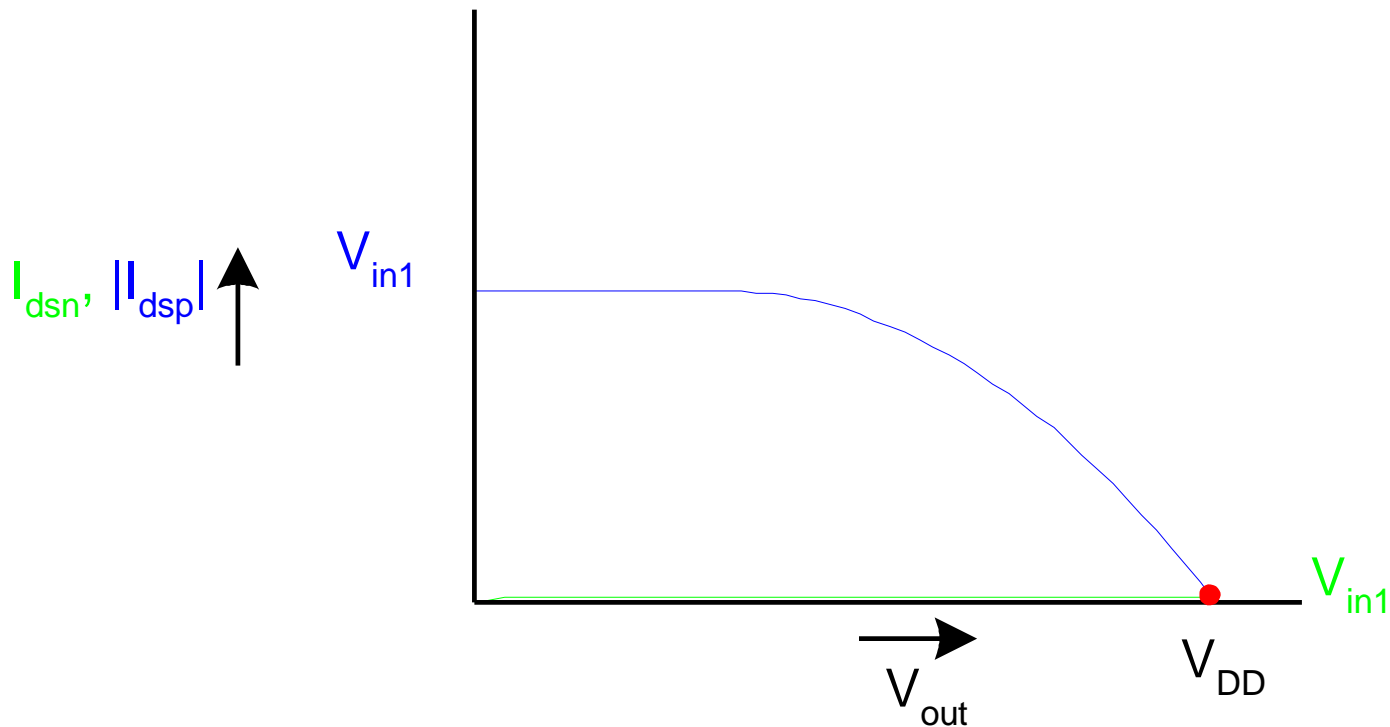
Load Line Analysis

- $V_{in} = 0$
- Transistor n is off



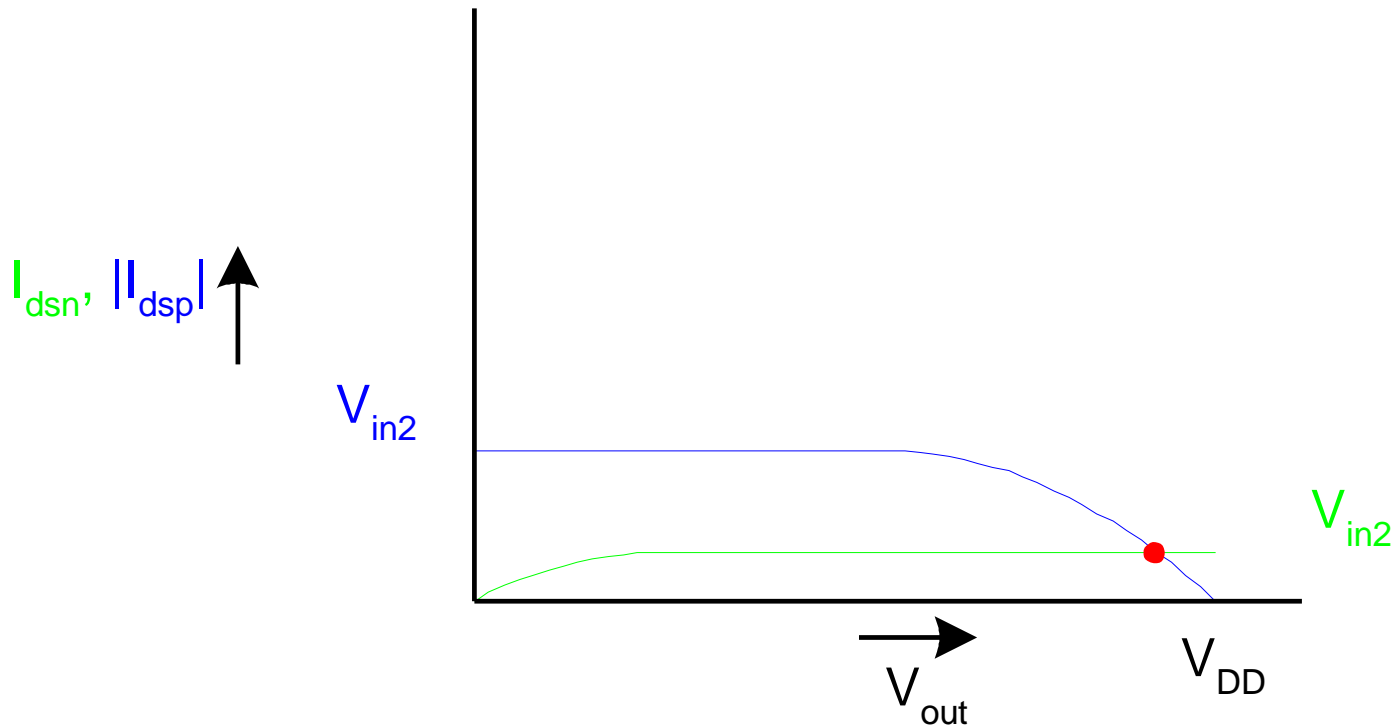
Load Line Analysis

- $V_{in} = 0.2V_{DD}$



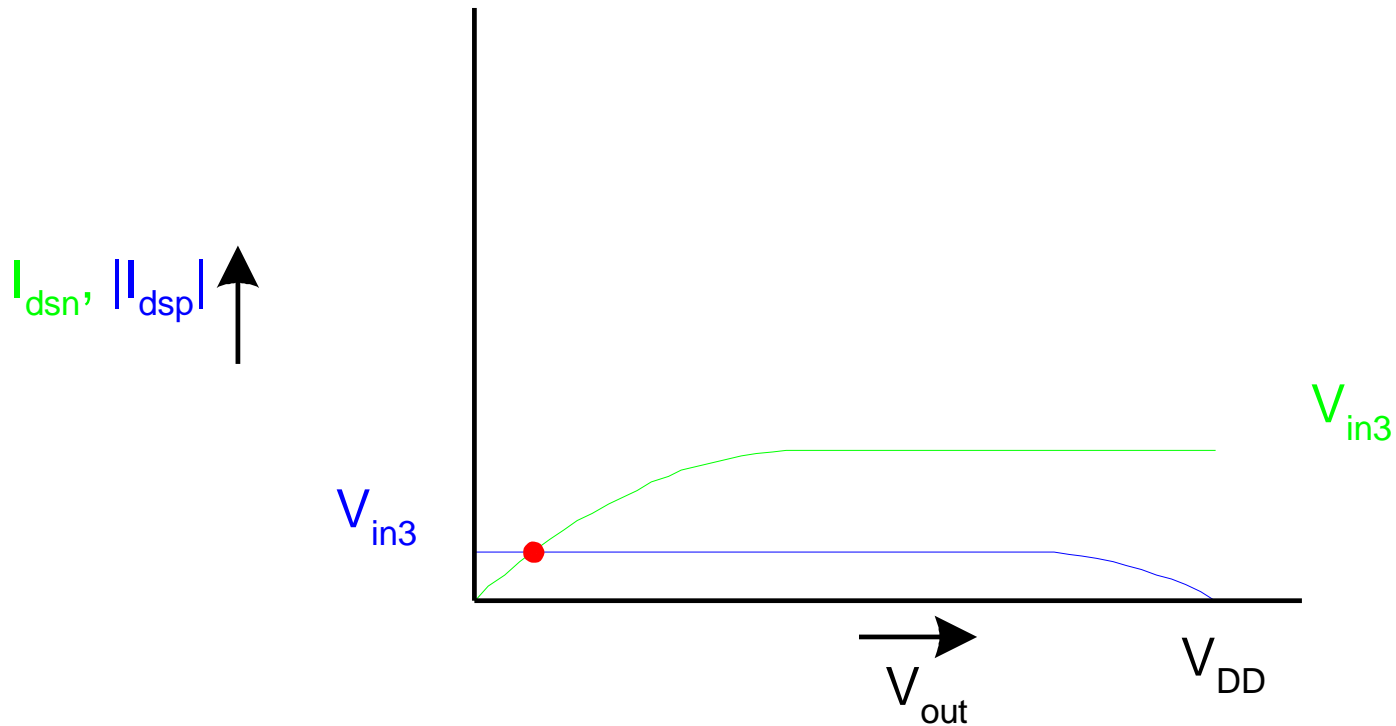
Load Line Analysis

- $V_{in} = 0.4V_{DD}$



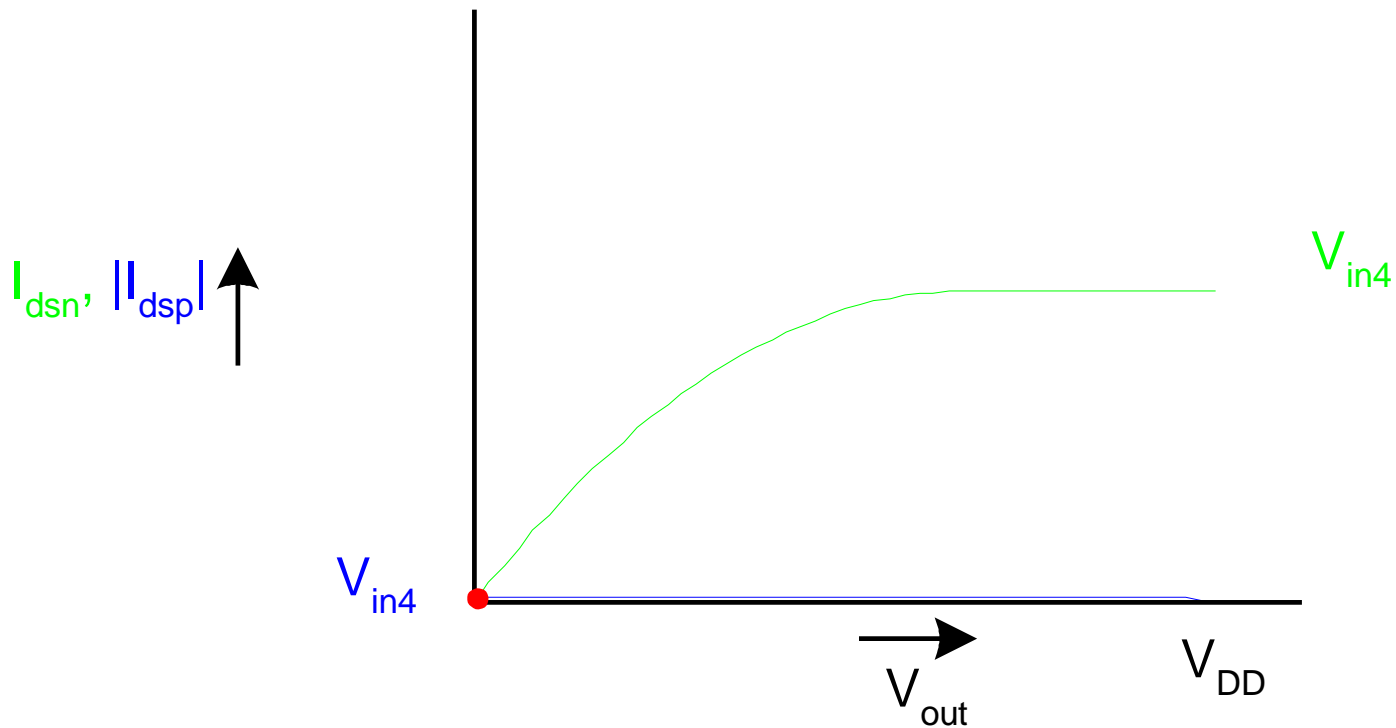
Load Line Analysis

- $V_{in} = 0.6V_{DD}$



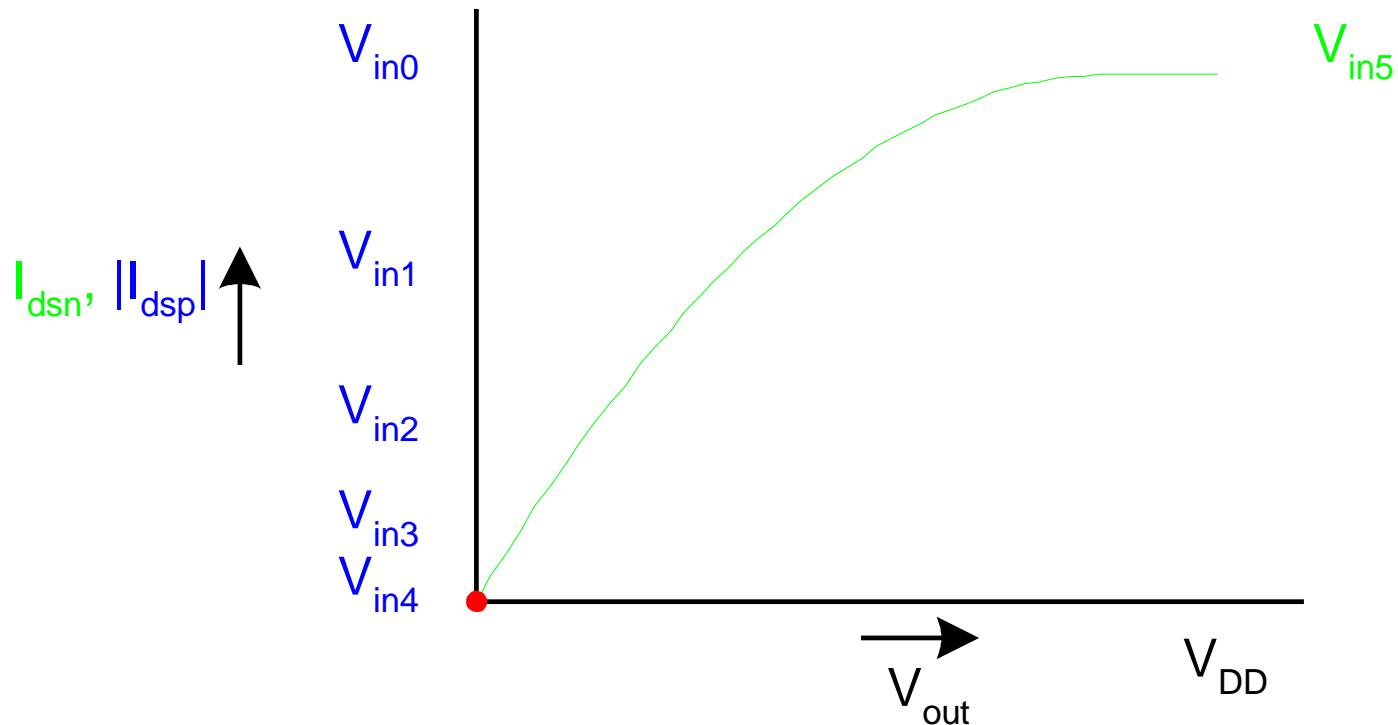
Load Line Analysis

- $V_{in} = 0.8V_{DD}$

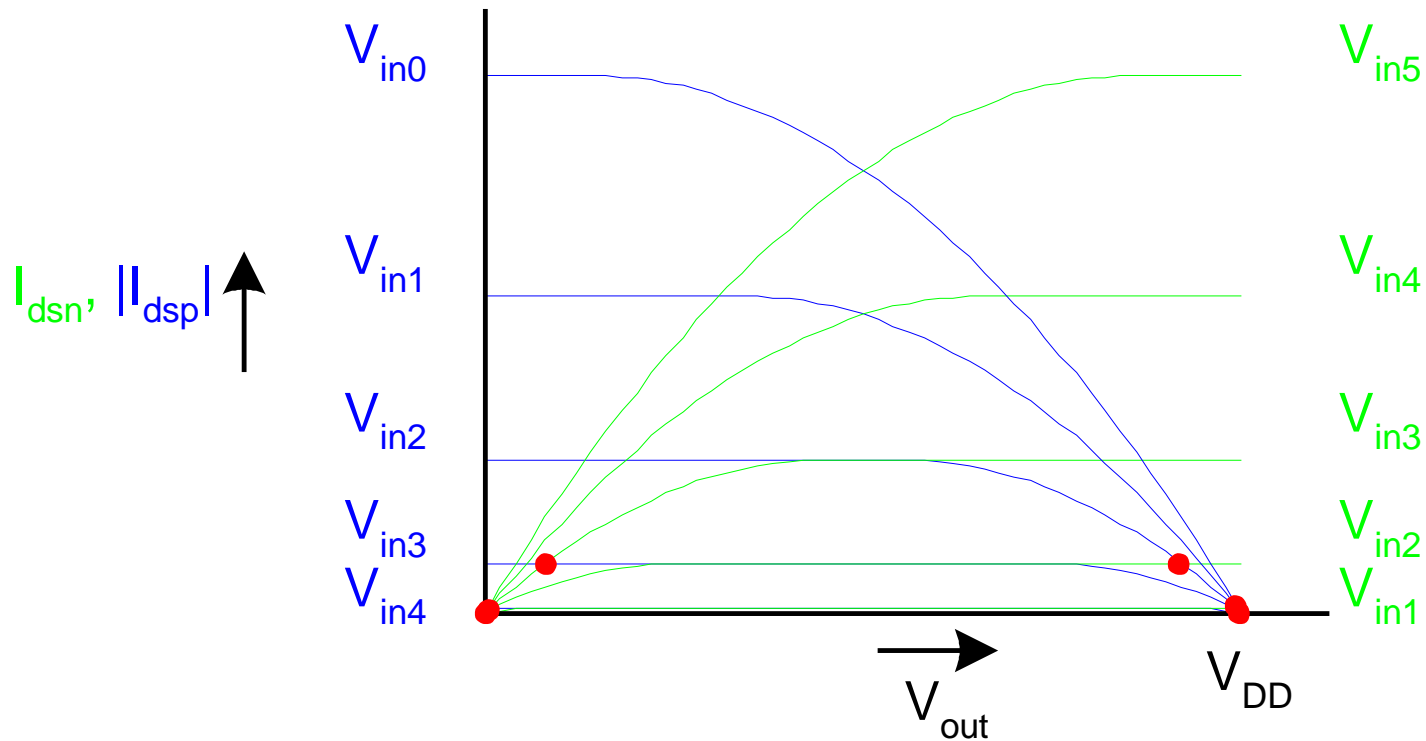


Load Line Analysis

- $V_{in} = V_{DD}$
- Transistor p is off

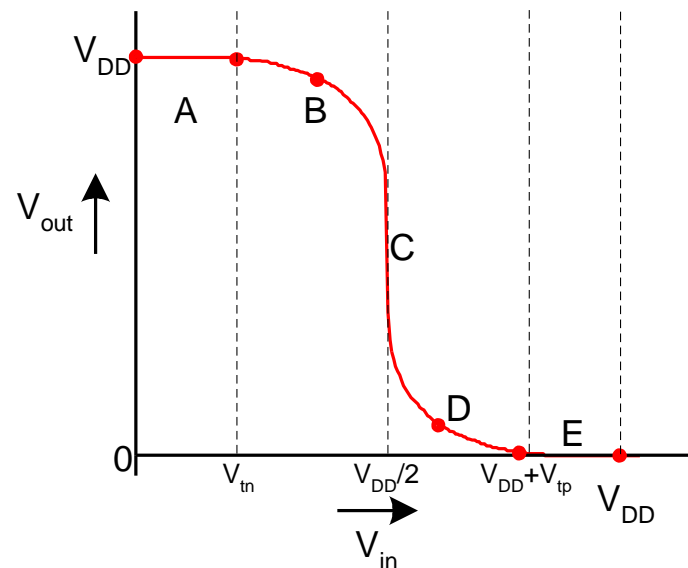
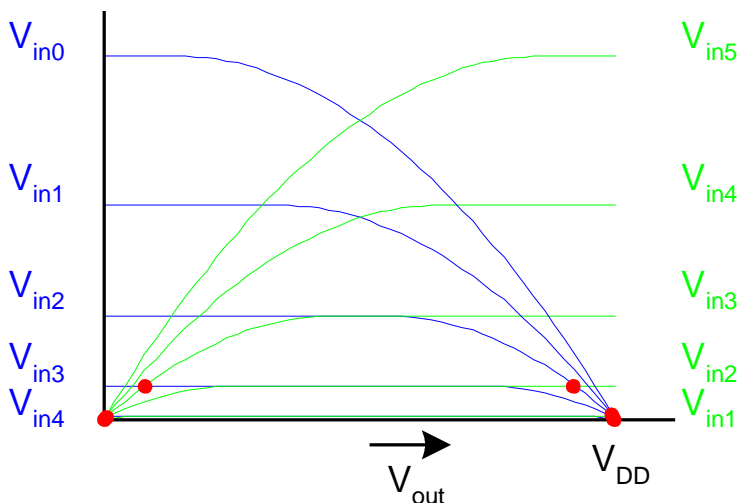


Load Line Summary



DC Transfer Curve

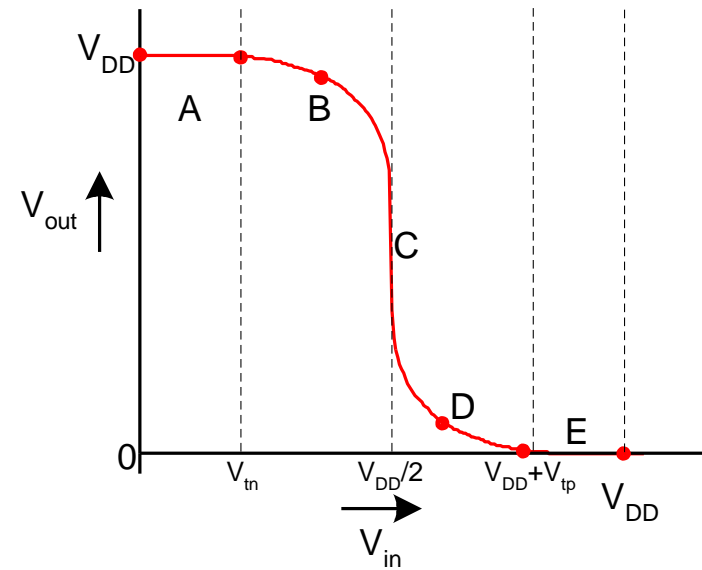
- Transcribe points onto V_{in} vs. V_{out} plot



Operating Regions

- Revisit transistor operating regions

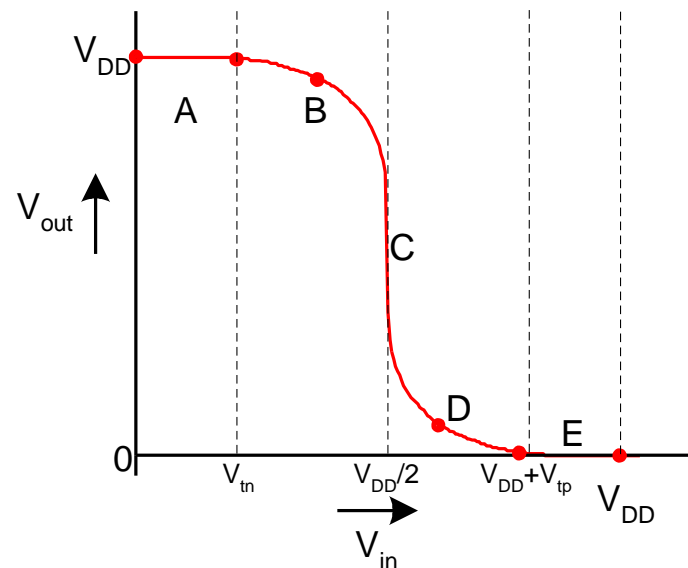
Region	nMOS	pMOS
A		
B		
C		
D		
E		



Operating Regions

- Revisit transistor operating regions

Region	nMOS	pMOS
A	Cutoff	Linear
B	Saturation	Linear
C	Saturation	Saturation
D	Linear	Saturation
E	Linear	Cutoff



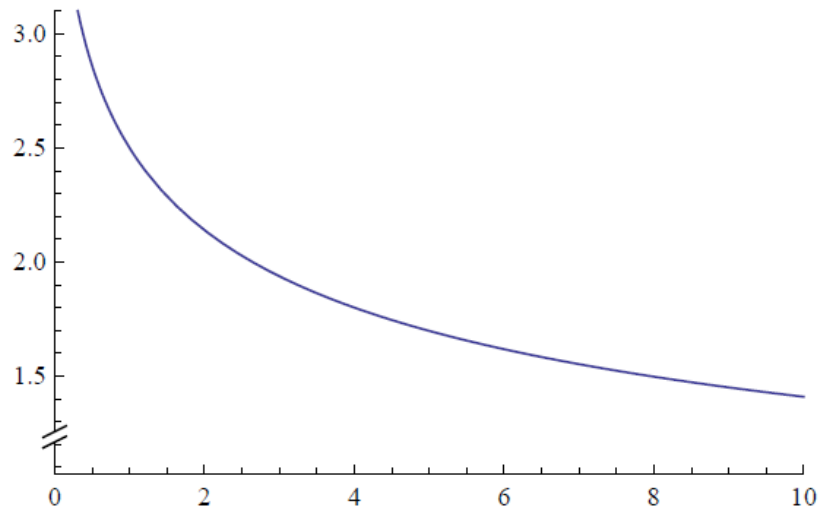
Dependencia del umbral con la relación de aspecto

```
Plot[(4.6 + 0.4 Sqrt[y])/(1 + Sqrt[y]), {y, 0, 10}]
```

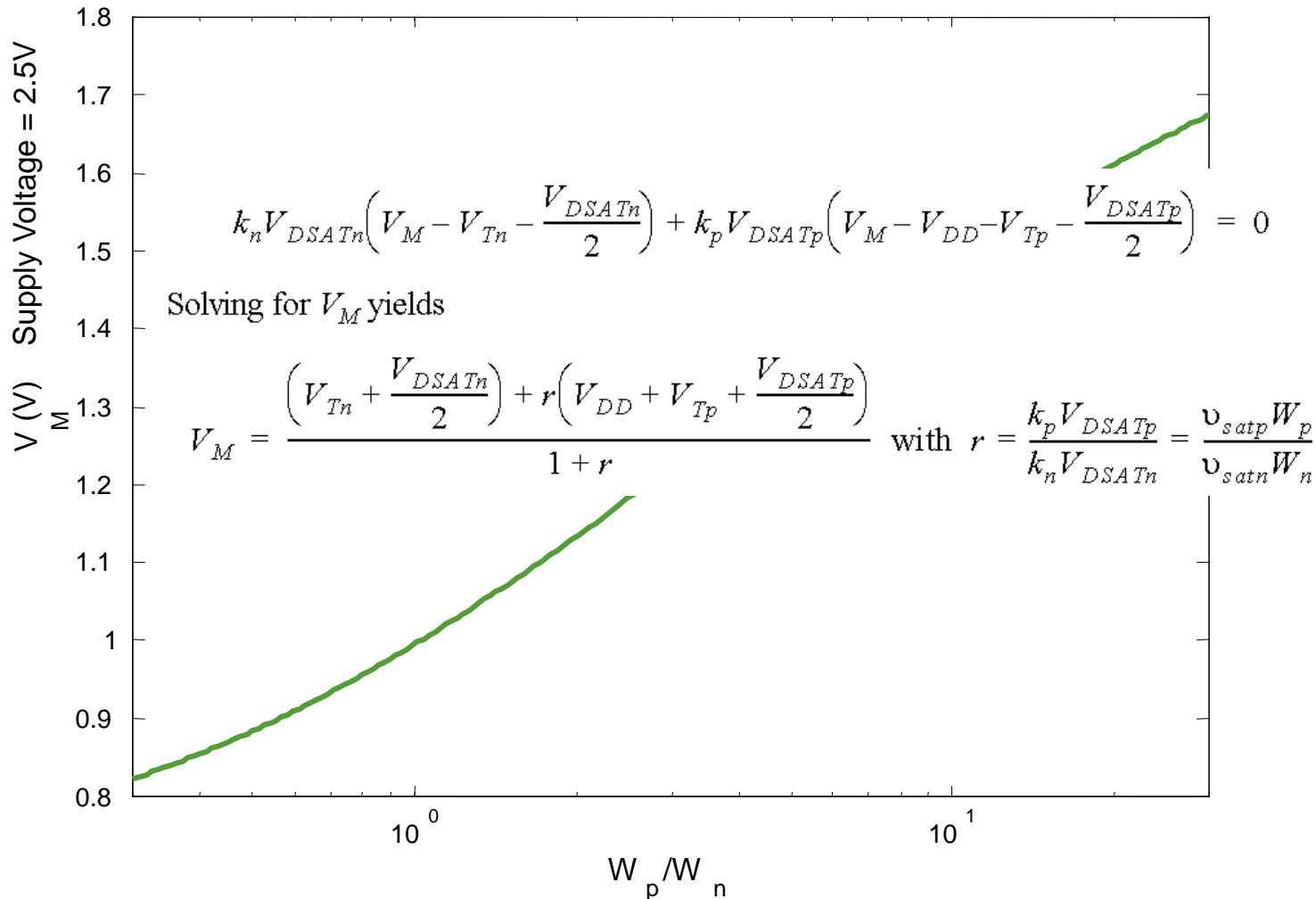
Input interpretation:

plot	$\frac{4.6 + 0.4 \sqrt{y}}{1 + \sqrt{y}}$	$y = 0 \text{ to } 10$
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Plot:

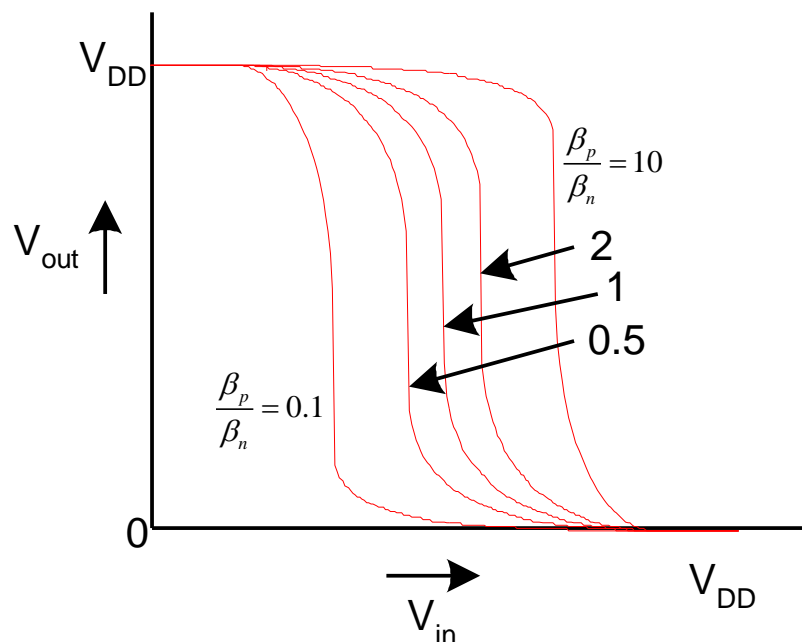


Switching Threshold as a function of Transistor Ratio



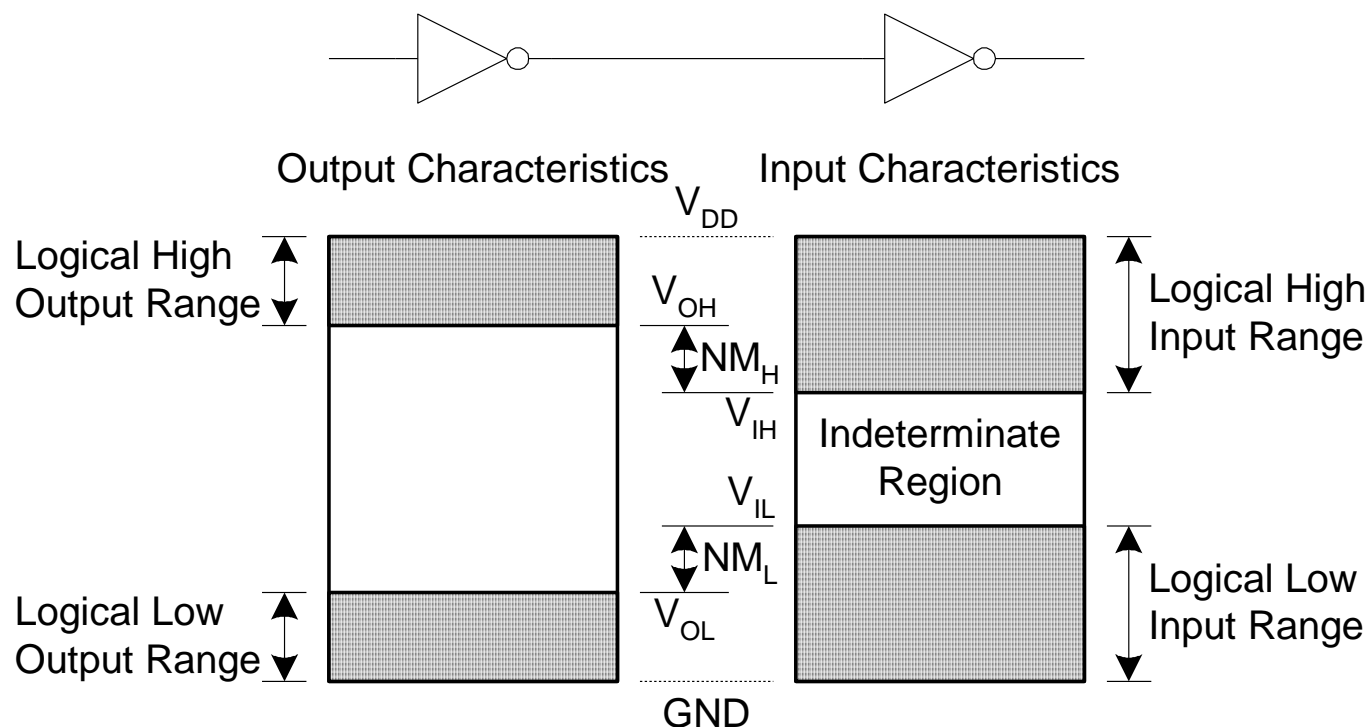
Beta Ratio

- If $\beta_p / \beta_n \neq 1$, switching point will move from $V_{DD}/2$
- Called *skewed gate*
- Other gates: collapse into equivalent inverter



Noise Margins

- How much noise can a gate input see before it does not recognize the input?

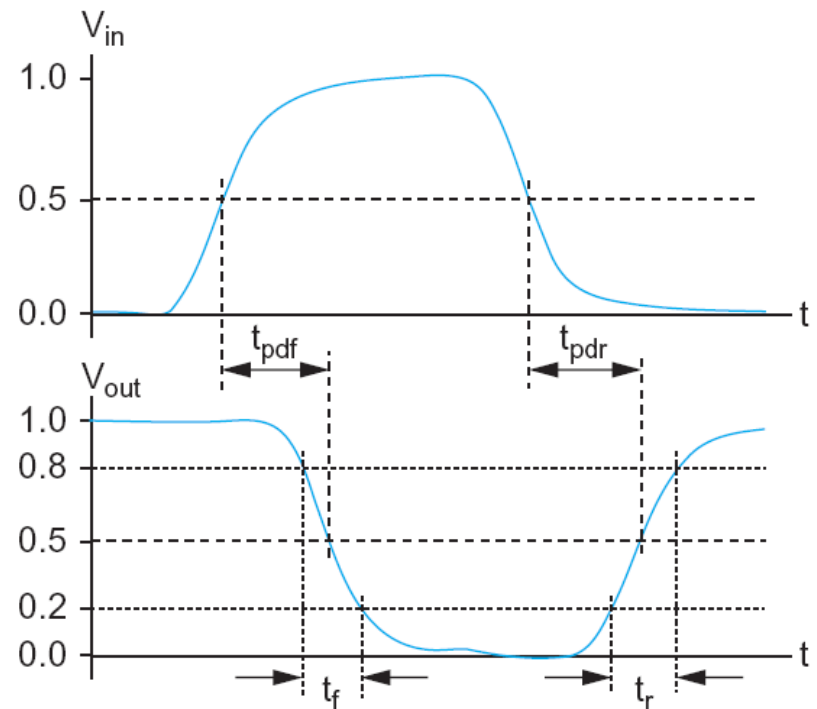


Transient Response

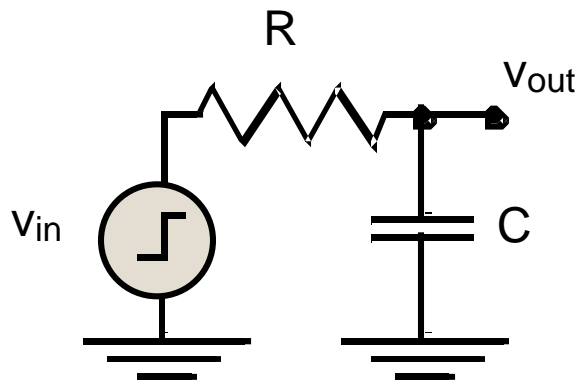
- *DC analysis* tells us V_{out} if V_{in} is constant
- *Transient analysis* tells us $V_{out}(t)$ if $V_{in}(t)$ changes
 - Requires solving differential equations
- Input is usually considered to be a step or ramp
 - From 0 to V_{DD} or vice versa

Delay Definitions

- t_{pdr} : *rising propagation delay*
 - From input to rising output crossing $V_{DD}/2$
- t_{pdf} : *falling propagation delay*
 - From input to falling output crossing $V_{DD}/2$
- t_{pd} : *average propagation delay*
 - $t_{pd} = (t_{pdr} + t_{pdf})/2$
- t_r : *rise time*
 - From output crossing $0.2 V_{DD}$ to $0.8 V_{DD}$
- t_f : *fall time*
 - From output crossing $0.8 V_{DD}$ to $0.2 V_{DD}$



A First-Order RC Network



$$V(t) = \frac{Q(t)}{C} = \frac{1}{C} \int_{t_0}^t I(\tau) d\tau + V(t_0)$$

$$I(t) = \frac{dQ(t)}{dt} = C \frac{dV(t)}{dt}$$

$$v_{out}(t) = (1 - e^{-t/\tau}) V$$

$$t_p = \ln(2) \tau = 0.69 RC$$

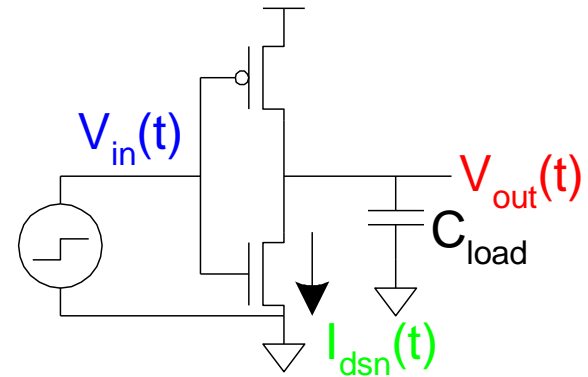
Inverter Step Response

- Ex: find step response of inverter driving load cap

$$V_{in}(t) =$$

$$V_{out}(t < t_0) =$$

$$\frac{dV_{out}(t)}{dt} =$$



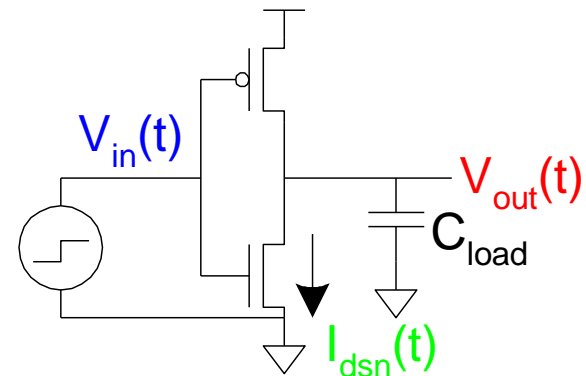
Inverter Step Response

- Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) =$$

$$\frac{dV_{out}(t)}{dt} =$$



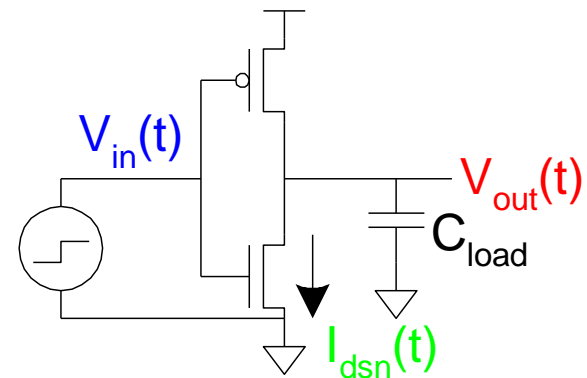
Inverter Step Response

- Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) = V_{DD}$$

$$\frac{dV_{out}(t)}{dt} =$$



Inverter Step Response

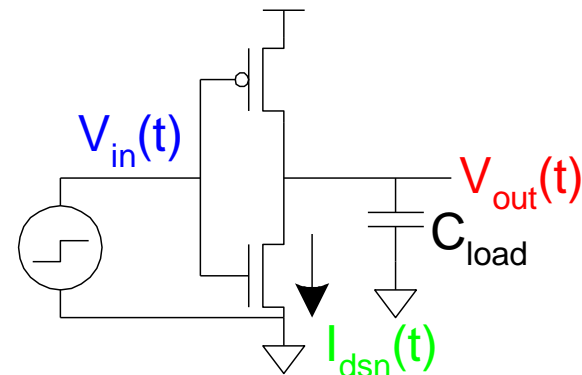
- Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) = V_{DD}$$

$$\frac{dV_{out}(t)}{dt} = -\frac{I_{dsn}(t)}{C_{load}}$$

$$I_{dsn}(t) = \begin{cases} & t \leq t_0 \\ V_{out} > V_{DD} - V_t \\ V_{out} < V_{DD} - V_t \end{cases}$$



Inverter Step Response

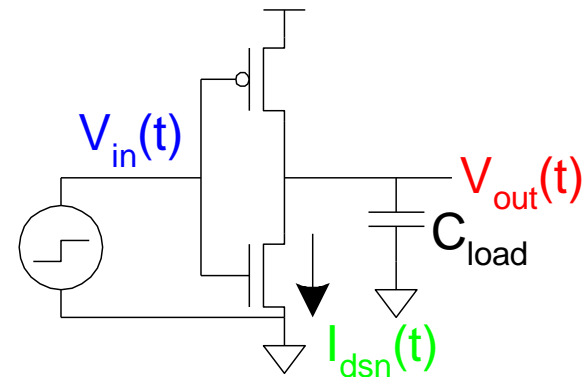
- Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) = V_{DD}$$

$$\frac{dV_{out}(t)}{dt} = -\frac{I_{dsn}(t)}{C_{load}}$$

$$I_{dsn}(t) = \begin{cases} 0 & t \leq t_0 \\ \frac{\beta}{2}(V_{DD} - V)^2 & V_{out} > V_{DD} - V_t \\ \beta\left(V_{DD} - V_t - \frac{V_{out}(t)}{2}\right)V_{out}(t) & V_{out} < V_{DD} - V_t \end{cases}$$



$$I(t) = \frac{dQ}{dt} = C \frac{dV}{dt}$$

Inverter Step Response

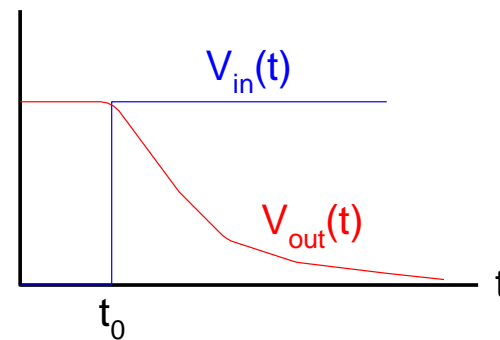
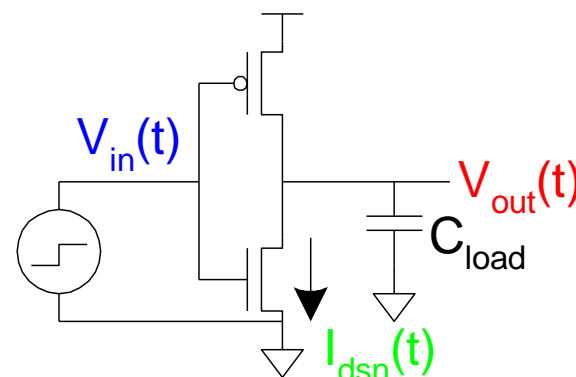
- Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) = V_{DD}$$

$$\frac{dV_{out}(t)}{dt} = -\frac{I_{dsn}(t)}{C_{load}}$$

$$I_{dsn}(t) = \begin{cases} 0 & t \leq t_0 \\ \frac{\beta}{2}(V_{DD} - V)^2 & V_{out} > V_{DD} - V_t \\ \beta \left(V_{DD} - V_t - \frac{V_{out}(t)}{2} \right) V_{out}(t) & V_{out} < V_{DD} - V_t \end{cases}$$



Average Currents Method

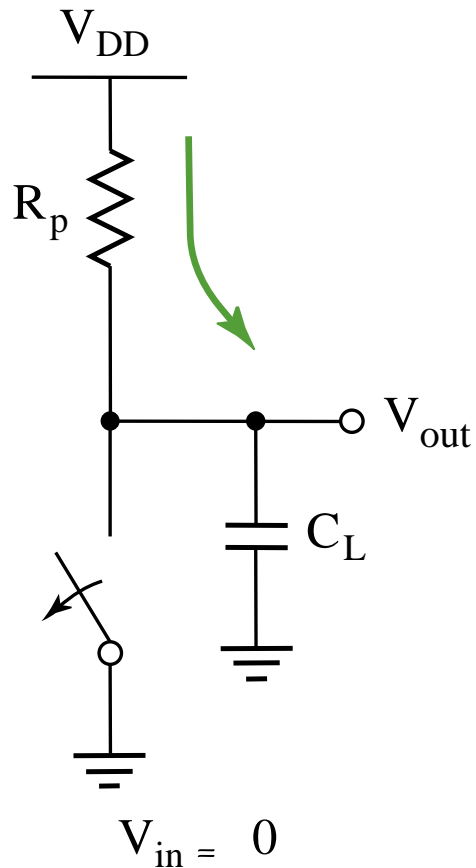
- Take the average of the current from the values at the extremes of the interval

$$t = \frac{C \cdot \Delta V}{I_{avg}}$$

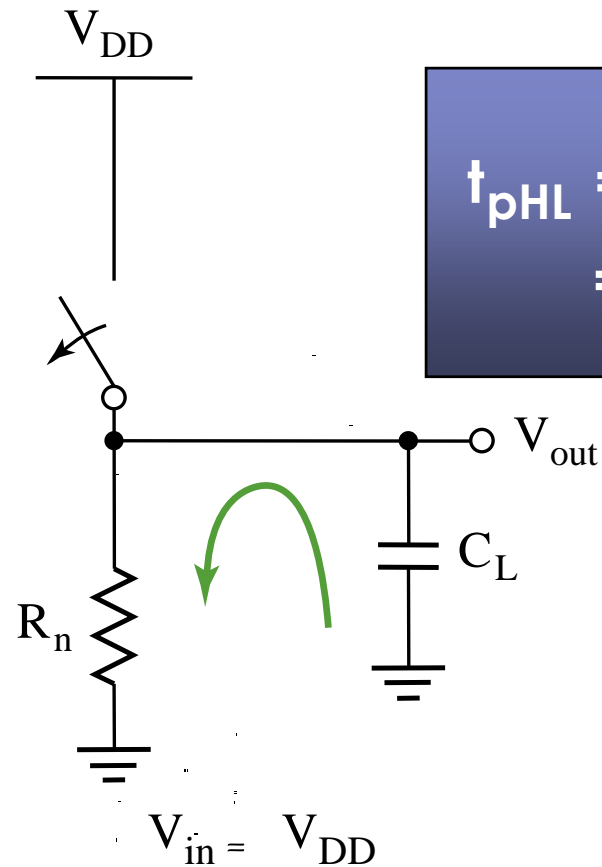
$$t_{PHL} = \frac{C_{load} \cdot \Delta V_{HL}}{I_{avg,HL}} = \frac{C_{load} (V_{OH} - V_{50\%})}{\frac{I_{V=V_{OH}} + I_{V=V_{50\%}}}{2}}$$

$$t_{PLH} = \frac{C_{load} \cdot \Delta V_{LH}}{I_{avg,LH}} = \frac{C_{load} (V_{50\%} - V_{OL})}{\frac{I_{V=V_{OL}} + I_{V=V_{50\%}}}{2}}$$

CMOS Inverter: Transient Response



(a) Low-to-high



(b) High-to-low

$$t_{pHL} = f(R_{on} \cdot C_L) \\ = 0.69 R_{on} C_L$$

Pseudo-nMOS

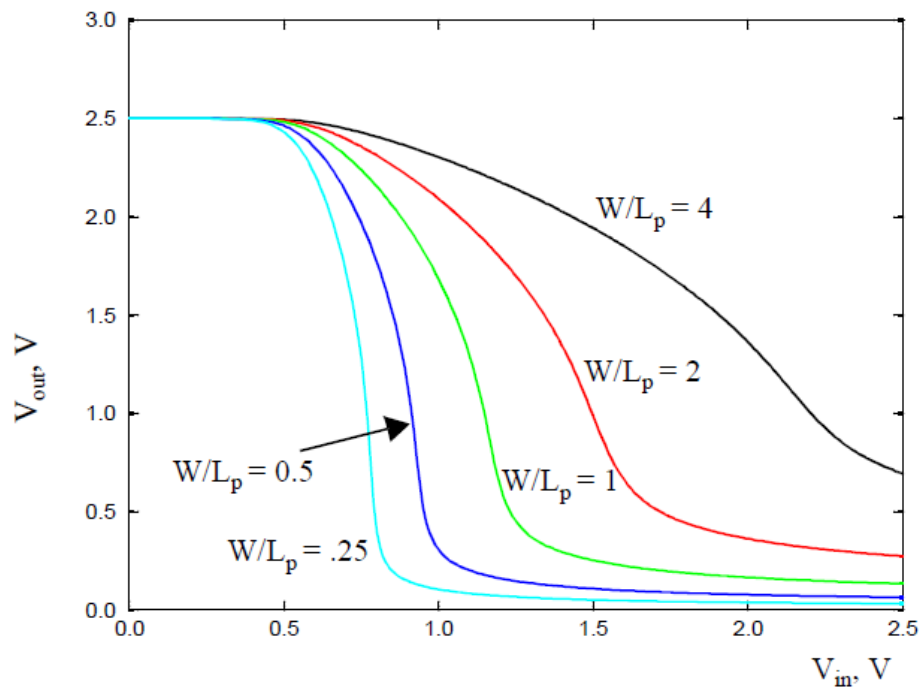


Figure 6.28 Voltage-transfer curves of the pseudo-NMOS inverter as a function of the PMOS size.